Teaching AI for Math

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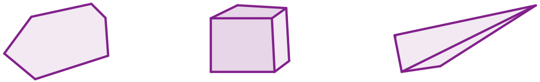
About Me

- Applications of polyhedral geometry: (discrete) optimization, machine learning (expressivity, explainability, learning structure), game theory, combinatorics



- Computer as tool
  - `polymake` developer (mostly during my PhD)
  - Using Oscar, Sage, . . . in research process
  - Experimenting with use of LLMs
  - Basics of polyhedral geometry in Lean

Workshop: ‘Polyhedra in Lean’ (August 24 – September 4, 2026 at FU Berlin)

Idea of the course

- Learn mathematics
- Learn how to use computers
- Get a feeling for the power of AI

Target group: maths students around 5th semester

- basics of mathematical work is required
- the course is meant to motivate students to attend further advanced courses

Learning goals

- overview of various techniques in mathematical research and how computers can help with them
- basic familiarity with technical tools (Lean, SAT-solvers, ...)
- communication about mathematical research

Conceptual blocks

- Mathematics as language: formalization, constructive mathematics, set theory vs type theory
- Reasoning: (automated) logical deduction
- Computing objects for existence and counterexamples: use 'old' mathematical software like SAT solvers, MIP solvers, Groebner bases computations, ...
- Generating and learning from mathematical data
- Finding mathematical objects and proofs as a game: reinforcement learning
- LLMs: N-gram language models for intuition, experimenting with most recent models

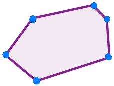
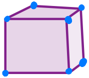
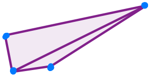
Structure of unit

- Introduction to the method
- historical or recent success stories
- show how to apply to ‘new’ problem

Mathematics as (formal) language: Lean

```
namespace Convexity

variable {R X Y V A : Type*}

open ConvexSpace

section Semiring

variable [Semiring R] [PartialOrder R] [IsStrictOrderedRing R]
variable [ConvexSpace R X]

variable (R) in
/-- A set is a polytope if it is the convex hull of finitely many points. -/
def IsPolytope (s : Set X) : Prop := ∃ t : Finset X, s = convexHull R t

end Semiring
```

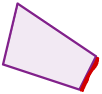
https://github.com/ooovi/Polyhedral/blob/main/Polyhedral/Mathlib/Geometry/Convex/ConvexSpace/Polytope/Basic.lean

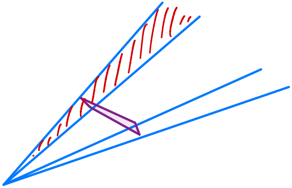




Mathematics as (formal) language: Lean

```
/-- Faces of polytopes are polytopes. -/
theorem face_isPolytope (hCfg : IsPolytope R (C : Set A)) (F : C.Face) :
    IsPolytope R (F : Set A) := by
  letI hom : IsHomogenization R A (CanonicalHomogenization R A) := inferInstance
  letI := IsModuleConvexSpace.ofAddTorsor (R := R) (V := (CanonicalHomogenization R A))
  have homC := IsPolytope.homogenize_FG (W := (CanonicalHomogenization R A)) hCfg
  have homF := hom.homogenize_isFaceOf F.isFaceOf
  have := PointedCone.IsFaceOf.fg homC homF
  convert FG.dehomogenize_isPolytope this (fun _ a b ↦ weight_pos_of_mem_homogenize a b)
  simp [dehomogenize_homogenize]
```

https://github.com/ooovi/Polyhedral/blob/main/Polyhedral/Mathlib/Geometry/Convex/ConvexSpace/Polytope/Face.lean





Further take-away: mathlib as **the** source of mathematical basics

Constructive mathematics: proving vs computing

Example: There exist irrational numbers x and y such that x^y is rational.
(Dov Jarden, 1953)

Non-Constructive

Constructive

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Non-Constructive

consider $\sqrt{2}^{\sqrt{2}}$ $\sqrt{2}$

Constructive

Consider $\sqrt{2}$ and $\log_2(9)$

Constructive mathematics: proving vs computing

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- Further reading: Constructive Mathematics and Computer Programming (Martin-Löf 1982)

General deduction

Home › Journal of Automated Reasoning › Article

A Deductive Database Approach to Automated Geometry Theorem Proving and Discovering

Published: October 2000

Volume 25, pages 219–246 (2000) Cite this article

Save article

Shang-Ching Chou, Xiao-Shan Gao & Jing-Zhong Zhang

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https://doi.org/10.1023/A:1006171315513

Article | Open access | Published: 17 January 2024

Solving olympiad geometry without human demonstrations

Trieu H. Trinh, Yuhuai Wu, Quoc V. Le, He He & Thang Luong

Nature **625**, 476–482 (2024) | Cite this article

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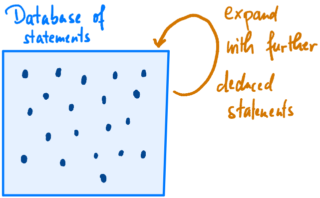
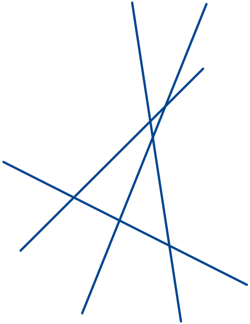
407k Accesses | **422** Citations | **1005** Altmetric | Metrics

https://doi.org/10.1038/s41586-023-06747-5





SAT solvers and LLMs

- Counterexamples to two conjectures about matroids

  (Matt Larson, 2026) `https://arxiv.org/abs/2607.02208`

  Idea of one of the conjectures: strong symmetric basis exchange properties for pairs of bases

How Example 1.4 was found. After checking that there are no counterexamples to Conjecture 1.2 in rank at most 4, I prompted ChatGPT 5.5 Pro to try to check Conjecture 1.2 in small rank using a SAT solver. It implemented an approach and claims to have used this to prove Conjecture 1.2 in rank 5 and 6[1]. I checked Conjecture 1.2 for many random matroids of ranks $7, 8, 9$, and 10 which are realizable over $\mathbb{F}_2, \mathbb{F}_3, \mathbb{F}_4$, or $\mathbb{F}_5$. I prompted ChatGPT to look for a counterexample around 20 times; see here for a typical chat. I then prompted it to look for ternary matroids of ranks $7, 8, 9$, and 10 and then to look for binary matroids of ranks 7 and 8. When I prompted it to look at binary matroids of rank 9, it found a matroid isomorphic to the matroid in Example 1.4; see here.

Another description of the matroid in Example 1.4 is that it is isomorphic to the binary matroid realized by the row span of the 9×18 matrix $[I|A]$, where I is the 9×9 identity matrix and

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

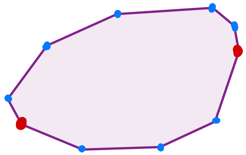
[1] I have not verified the correctness of this computation.

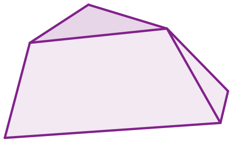
SAT solvers and LLMs

- Recently: counterexample to ‘Jacobian Conjecture’
  - Alpöge (2026) with Fable: `https://x.com/__alpoge__/status/2079028340955197566`
  - Zihan Zhang (2026) some mathematical implications: `https://zzhang-iu.github.io/papers/direct-consequences-jacobian/`

Polyhedral computations and LLMs

- A counterexample to the Hirsch Conjecture
  (Paco Santos, 2012; Annals of Mathematics)
  the graph of a d-dimensional polytope with n facets **can** have a diameter greater than $n - d$:
  explicit construction of a 43-dimensional polytope with 86 facets and diameter at least 44.





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- Advancing Geometry with AI: Multi-agent Generation of Polytopes `https://arxiv.org/abs/2502.05199`
  Drums of high width
  `https://londmathsoc.onlinelibrary.wiley.com/doi/abs/10.1112/plms.70031`

Take aways from earlier versions of the seminar

- Well-prepared technical setup important for students to actually use the tools
- Different aspects of difficulty: mathematical understanding of the specific questions, technical affinity, combining different approaches

Recap conceptual blocks

- Mathematics as **language**: formalization, constructive mathematics, set theory vs type theory
- Reasoning: (automated) logical **deduction**
- **Computing objects** for existence and counterexamples: use ‘old’ mathematical software like SAT solvers, MIP solvers, Groebner bases computations, ...
- Generating and learning from **mathematical data**
- Finding mathematical objects and **proofs as a game**: reinforcement learning
- LLMs: N-gram language models for intuition, **experimenting** with most recent models

Further considerations

- Ethics
- Social aspects
- Sustainability